

## Designing an optical mechanics experiment

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**ABSTRACT:** An optical engineering experiment that combines optical science and the mechanics of materials is presented. The goal is to enhance stress concentration instruction, as well as engineering optics by incorporating optical experiments using caustics within the existing curricula. The proposed experimental method and demonstration are of fundamental value to mechanical engineering students as it introduces the students to an optical stress transducer. The proposed experiment creates a hands-on learning opportunity of engineering optics for students. Details of the theory and experimental setup needed for undergraduate civil and mechanical engineering students are presented. The optical bench and the material for the proposed optical mechanics experiment are economical. Therefore, the integration of this optical mechanics experiment adds a new dimension to a mechanical engineering laboratory.

### INTRODUCTION

Learning about the mechanics of materials can be enhanced through experimental demonstrations [1-3]. At the sophomore level, students in civil and mechanical engineering programs are introduced to the concepts of stress and strain in a solid body through the Strength of Materials course. In the same course and/or the Machine Design course, junior mechanical engineering students learn to get stress concentration factors (SCF) for practical problems from a chart.

The stress concentration theory usually deals with infinite members. Kirsch developed the theoretical stress distribution near a circular hole in an infinite elastic isotropic plate [4]. This theory predicts a SCF of 3.0 for the hole with the maximum tensile and compressive stresses being 0 and 90 degrees from the horizontal axis of the hole, respectively. In the field of stress concentrations, the limited established theory does not give an insight into the understanding of the development of stresses near a discontinuity. Thus, experimental work is required to enhance the learning stress concentrations [5]. Howland published the solution for the circular hole in a finite-width plate under uniaxial tension problem in 1930 [6]. In more recent years, experimental solutions have been obtained for a wide variety of hole shapes under different loading conditions as documented in References [7][8].

Feisel and Rosa discussed the role of the laboratory in engineering education [9]. It is important that the students visualise the nature of the quantities being computed. Therefore, the enhancement of the student's overall understanding of the concept of stress concentrations is discussed in this article. This is accomplished by utilising the experimental method of caustics. The determination of SCF is beyond the scope of this article. The design of an optical stress transducer experiment is presented.

### SETTING

Mechanical engineering students at Indiana University-Purdue University Fort Wayne learn the fundamentals of optics in a physics class: Electricity and Optics. They are introduced to geometrical and physical optics and most likely will not use the knowledge learned in the rest of the curriculum. The proposed experiments introduce the students to an experimental technique of caustics for the understanding of stress concentration effect in the vicinity of a hole in a plate. The objectives of the experiments that use specimens in uniaxial tension are to show the students the following:

- The effect of stress concentration in a member.
- Design an optical engineering experiment to measure the applied stress that is required in many practical engineering situations.
- An application of optics in civil and mechanical engineering.
- A limitation of the theory of elasticity.

## OPTICAL METHOD OF CAUSTICS

The emphasis of this article is on the balance between applied mathematics, engineering science and experiment. The optical method of caustics has proved to be a powerful experimental technique to measure stress intensity factor at a crack tip in static and dynamic fracture mechanics problems, for recent example [10]. Due to its high sensitivity to stress gradients, the author has used it in a mechanics laboratory as a tool to provide useful information about the intensity of the stress field in the vicinity of a hole. The optical method of caustics is a technique based on geometrical optics. The method is accurate, simple and economical because the optical bench has relatively few components. The accuracy of the proposed experiments derives from the fact that the physical stress models must obey the practical laws of physics.

The principle of the method is simple in concept. The formation of the caustic image is dependent on the stresses in a structural member or machine component. Therefore, it is an ideal method to be used for when there is a SCF, since high stress gradients produce large deflection of the light rays and an image with distinguishing characteristics. The advantage of caustics relative to other optical experimental techniques is that the same equipment can be used in either, a reflection or a transmission arrangement.

### Strength of Materials

The optical method of caustics is a powerful method to study the development of SCF because of its high sensitivity to stress gradients. The change in thickness of the specimen in plane stress condition can be obtained from generalised Hook's law, in terms of the in-plane principle stresses  $\sigma_1$  and  $\sigma_2$ , as:

$$\delta d = -\frac{\nu}{d}(\sigma_1 + \sigma_2) d \quad (1)$$

The stress singularity of the elastic field is transformed into optical one represented by a highly illuminated surface that contains the necessary information for determining the applied stress in this study. If a specimen that contains a central hole is loaded, the state of stress in the vicinity of the hole is much higher than the stresses along the rest of the specimen. The sum of in-plane principal stresses in the vicinity of a hole is given by Kirsch's solution [4] as:

$$\sigma_1 + \sigma_2 = \frac{\sigma}{2} \left( 2 + \frac{4a^2}{r^2} \cos 2\theta \right) \quad (2)$$

where  $r$  and  $\theta$  are the polar coordinates with the origin at the centre of the hole. It is important to remember that the above equation is for infinite plate as it will be discussed in the results section.

### Optics

In the second general physics course, engineering students learn the fundamentals about light, images and diffraction. The method of caustics is based on the principles of geometrical optics. Since  $\sigma_1$  and  $\sigma_2$  vary with  $r$  and  $\theta$ , the front and back surfaces of the specimen deform. If a monochromatic and coherent light beam impinges on the face of transparent specimen, the deformed surfaces cause the light rays to deflect like a divergent lens. Upon transmitting and exit from the specimen, the rays are not parallel. If a screen is placed at distance  $Z_0$ , shown in Figure 1, downstream from the specimen, the light rays produce an interesting optical pattern, caustics. The deviation vector  $\mathbf{D}$  resulting from the light ray transmitted or reflected from the area very close to a hole in an optically isotropic medium is shown in Figure 1. The direction and magnitude of the deviation vector are correlated to the change in the optical path  $\delta s$  and it is given by Eikonal [11] equation as:

$$\mathbf{D} = Z_0 \text{grad } \delta s(r, \theta) \quad (3)$$

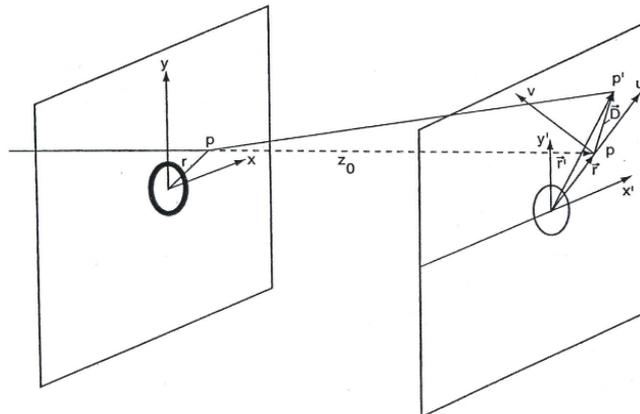


Figure 1: Development of the image equation.

Utilising the fundamentals of mirrors, the light's optical paths can be established. Consider the light ray AB that traverses the unstressed specimen as shown in Figure 2. In terms of the refractive index of the material, the initial optical path of the transmitted light is:

$$L_i = AB + nd + CD. \quad (4)$$

When a tensile stress is applied, both the thickness and refractive index of the material decrease. The final optical path of the transmitted light is equal to:

$$L_f = AB + \delta d + (n - \delta n) (d - \delta d) + CD \quad (5)$$

The change in the optical path ( $L_f - L_i$ ) of the light, neglecting higher order term, can be calculated as:

$$\delta s = (n - 1)\delta d + d\delta n \quad (6)$$

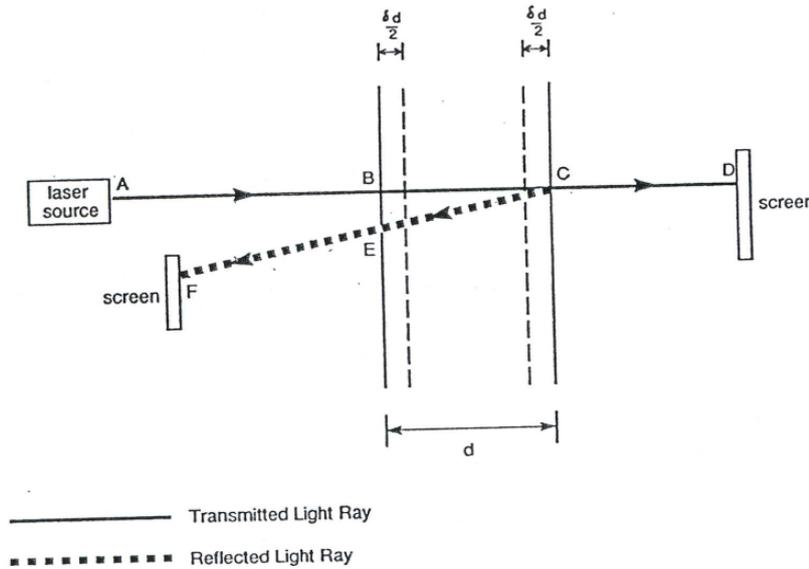


Figure 2: Optical path of light rays.

### Multidisciplinary

In this section, calculus, optics and strength of materials will be linked to establish the necessary optical mechanics relationships. The change in the refractive index of isotropic material due to the in-plane principal stresses is given by Maxwell's relationship [12] as:

$$\delta n = A (\sigma_1 + \sigma_2) \quad (7)$$

If one uses Equations (1) and (7) in Equations (6), then the change in the optical path becomes:

$$\delta s = d \left[ (1 - n) \frac{\nu}{E} + A \right] (\sigma_1 + \sigma_2) \quad (8)$$

Substituting Equations (2) and (8) into Equation (3), it yields:

$$\mathbf{D} = Z_o d \sigma \text{ grad} \left( 2 + \frac{4a^2}{r^2} \cos 2\theta \right) \left[ (1 - n) \frac{\nu}{E} + A \right] \quad (9)$$

The mapping of the specimen plane on the image plane is shown in Figure 1. The point p is placed in a local region near the hole. However, after propagating down the optical bench, the deflected light ray impinges the screen at point p whose location is given by the vector  $\mathbf{r}'$ . The image vector equation can be written as:

$$\mathbf{r}' = \mathbf{r} + \mathbf{D} \quad (10)$$

Substituting Equation (9) into the image Equation leads to:

$$x = r \cos \theta + 4Z_o d \sigma a^2 \left[ (1 - n) \frac{\nu}{E} + A \right] r^{-3} \cos 3\theta \quad (11a)$$

$$y = r \sin \theta + 4Z_o d \sigma a^2 \left[ (1 - n) \frac{\nu}{E} + A \right] r^{-3} \sin 3\theta \quad (11b)$$

The caustic curve is the line between the dark shadow zone and the bright band. Therefore, the caustic is a singular curve of the image Equation (10) and the necessary condition for the existence of such singularity is when Jacobian determinant is zero, thus:

$$\frac{\partial x}{\partial \theta} \frac{\partial y}{\partial r} - \frac{\partial x}{\partial r} \frac{\partial y}{\partial \theta} = 0 \quad (12)$$

From Equations (11) and (12), a relation for the initial curve (lens) radius of the caustic on the image plane is obtained:

$$R_i = (12 \sigma Z_0 c a^2)^{0.25} \quad (13)$$

where  $c$  is the optical constant. These relations show that the initial curve is a circle with radius  $R_i$ . Substituting the value for  $R_i$  in Equations (11), the corresponding image equations become:

$$x = R_i \left( \cos \theta + \frac{1}{3} \cos 3\theta \right) \quad (14a)$$

$$y = R_i \left( \sin \theta + \frac{1}{3} \sin 3\theta \right) \quad (14b)$$

The angle  $\theta$  varies between 0 and  $2\pi$ . Equation (14) generate a caustic curve which can be classified as a nephroid as shown in Figure 3.

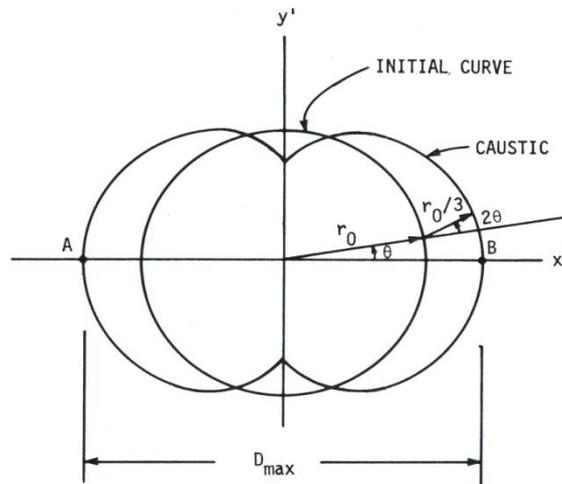


Figure 3: Theoretical initial curve and caustic.

### Experiment

An optical mechanics experiment was designed for the undergraduate civil/mechanical engineering laboratory. The experimental apparatus is relatively simple. The schematic and suggested arrangements of the optical system for the experimental transmitted caustics are shown in Figures 4 and 5, respectively. Briefly, a monochromatic and coherent light beam emitted from a point source He-Ne laser, which was widened by a spatial lens, impinges normally on the specimen. The light beam has to fulfil only one, very important requirement: it has to be parallel. To achieve this property, the light source must have the essential features of a point source.

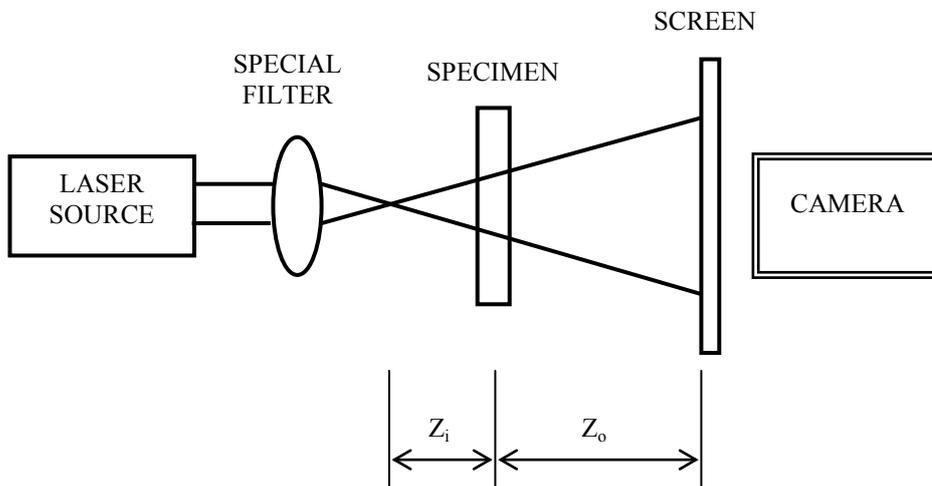


Figure 4: Schematic transmitted caustic setup.

Divergent light is used primarily to enlarge the caustic image. The direct recording of the caustic image is possible in transmission arrangements as well as in reflection arrangements. The rotation of the model produced a light beam that was not perpendicular to the specimen. This rotation created only a translation of the caustics without affecting the size, shape and relative position of the caustics. However, a rotation of the screen distorts the caustic image. Therefore, the screen should be always parallel to the model. The live caustic image can be captured by a camera.

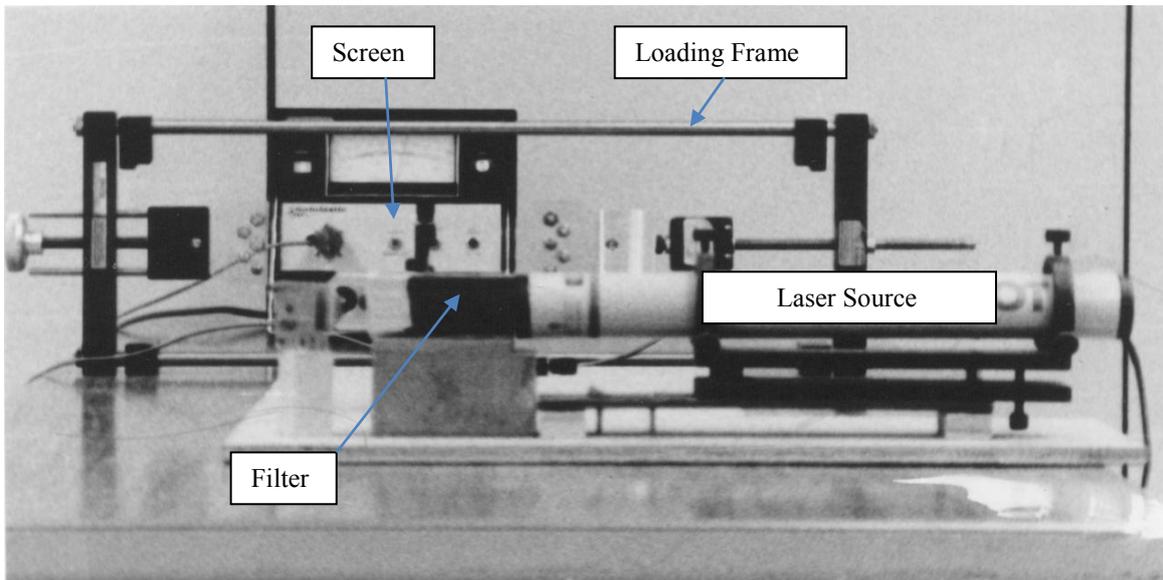


Figure 5: Experimental caustic setup.

#### Specimens and Image

The material used is polymethyl Methacrylate (PMMA, Plexiglas) because it has the advantage of being a mechanically and optically isotropic material. Models with the dimensions shown in Figure 6 were used. A small drill was used to bore holes in the models slowly. No residual stresses were noticed in the experiments. The residual stress can be detected from the small pseudo caustic it produces. All models were taken from the same Plexiglas sheet.

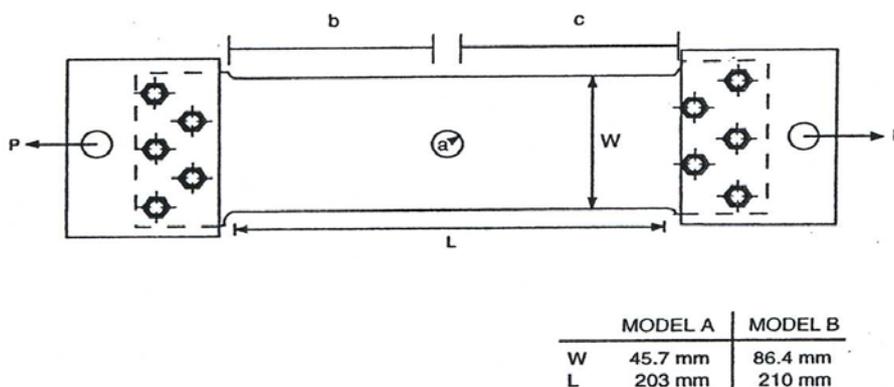


Figure 6: Test specimens.

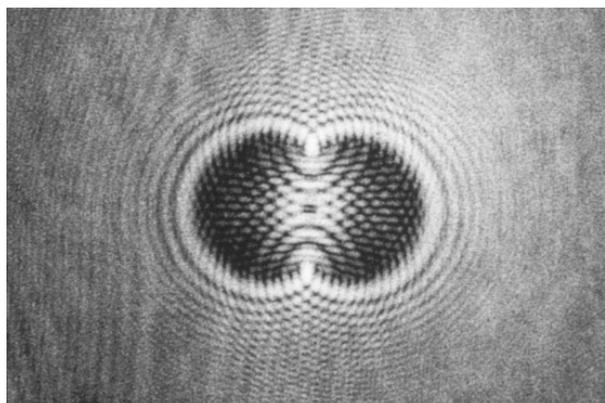


Figure 7: Caustic image.

The observed caustic image is shown in Figure 7. The pattern of regions is related to stress concentration as it describes the gradient of the sum of the in-plane stresses, Equation (2). The gray regions correspond to locations where the gradient of  $(\sigma_1 + \sigma_2)$  is small and the parallel rays through the transparent specimen with very small deflection. The dark areas give the shadow spot and are the result of the deflection of the light rays from this local area on the screen. On the other hand, the light regions are due to the added intensity of the normal rays in addition to those deflected rays which impinge on this area of the screen.

Relation (13) shows that the radius, which defines the envelope of the highly stressed zone of the specimen, is constant. Within the framework of linear elasticity, it should be observed that the deformed shape of the specimen surface near the hole (stress concentration) is proportional to the applied stress. Thus, the light patterns obtained from the specimen surface near the hole provide a direct measure of the applied stress.

## RESULTS

The maximum diameter of the caustic and the radius  $R_i$  of the initial curve are shown in Figure 3. The maximum diameter,  $D_{max}$  is related to  $R_i$  by Reference [13]:

$$D_{max} = 2.67 R_i \quad (15)$$

A good exercise for the student is to derive and verify the above relationship. Engineers are required to determine the actual stresses that are transmitted to a structural member or a machine element in many practical design and analysis situations. The interaction between members and elements, as well as the effect of assembly stresses necessitate the performance of this task experimentally. The need for the use of the method of caustics to measure the applied load in polymeric members is discussed in Reference [14]. Substituting Equation (15) into the initial curve equation leads to:

$$\sigma = \frac{(D_{max})^4}{610 Z_o c a^2 d M^2} \quad (16)$$

Equation (16) represents the basic relationship underlying the proposed optical method of measuring the stress. It is clear that the applied axial stress can be determined simply by measuring the maximum diameter of the image since  $d$ ,  $a$ ,  $c$ ,  $Z_o$ , and  $M$  are known parameters in a typical experiment. This experimentally applied stress is compared to the known applied stress ( $\sigma = P/A$ ). Each model was subjected to five different loads and some of the results obtained are shown in Table 1. The percentage difference between the actual and experimental stress is greater than eight when the size of the hole is increased. This is because Equation (2) is based on an infinity wide plate solution and the finite width has a significant effect on the results.

Table1: Comparison between actual and experimental results.

a/W	Applied Stress MPa	Optical Stress MPa	% Error
0.010	8.81	8.72	-1.02
0.018	7.33	7.14	-2.60
0.025	8.43	8.38	-0.59
0.031	6.28	6.17	-1.75
0.037	7.12	7.20	+1.12
0.041	5.63	5.71	+1.40
0.050	4.69	5.13	+8.57
0.056	6.04	6.93	+12.84
0.065	6.30	7.20	+10.10
0.071	7.00	7.62	+9.84
0.074	7.00	7.80	+10.81

By observing the caustic image, one can determine the region that is affected by the presence of stress concentration. The difference between the experimental and actual stresses is due to the limitation of the theory of elasticity resulting from the infinite plate solution. The effect of the hole size upon the determination of the calculated applied stress can be investigated by varying the ratio of the hole size  $a$  to the plate width  $W$ . The students learn the effect of the interaction of the finite width of the plate and the hole. At this stage, they can establish the validity of Equation (2) in regard to the infinity issue. Thus, one can design caustic experiments with specified magnification factor and valid  $a/W$  ratio to measure a physical quantity.

Theoretically, the relevant caustic line should be defined by the transition from the dark inner region to the bright rim of the caustic pattern. However, due to the light diffraction effects, the caustic rim will have a band shape rather than the theoretical fine line as shown in Figure 7. One should consider the inner diameter, outer diameter and the middle diameter of the caustic image in the calculations to appreciate the effects of engineering diffraction. The middle diameter is the ballpark average diameter and not the mean. The results in the table are based on the inner caustic

diameter. The percentage error based on the three diameters is shown in Figure 8. This hands-on experience helps the understanding of many abstract concepts of optics and its relation to engineering.

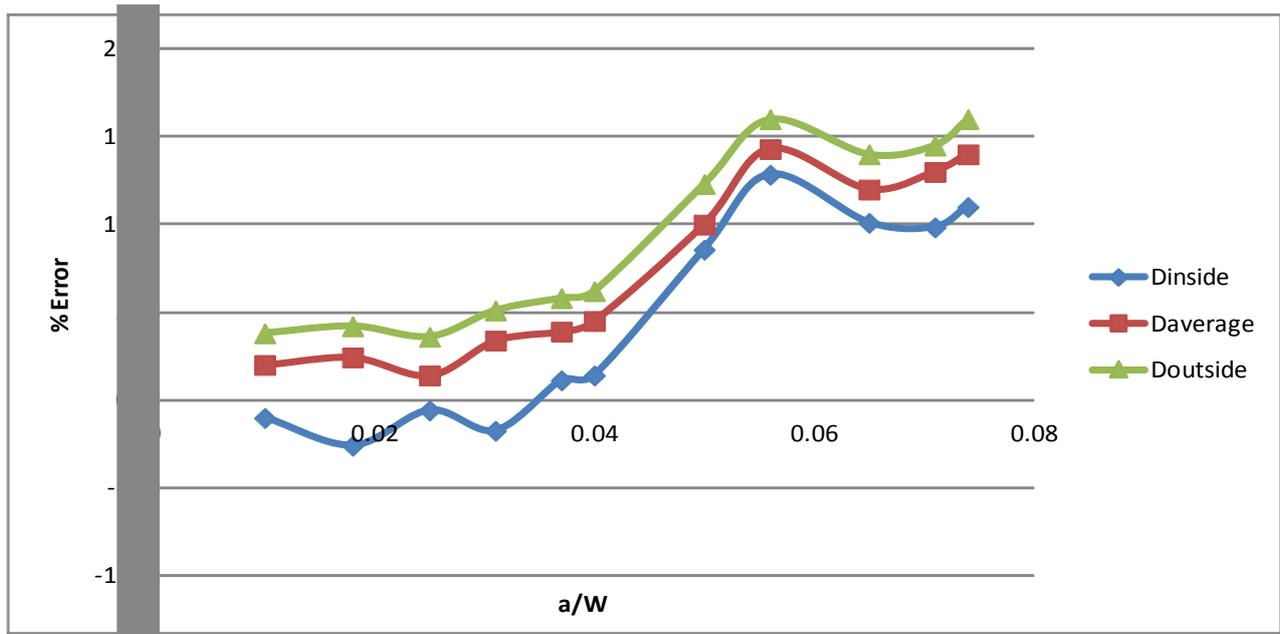


Figure 8: Percentage error versus a/W.

## CONCLUSIONS

The caustic experiment described in this article is a valuable addition to any undergraduate mechanics laboratory. It enhances the learning of some of the engineering optics stress concentration fundamentals, and limitations of the theory of elasticity. The optical bench has relatively few components and the material used is readily available at very low cost. Thus, the experiment is simple and economical. Details of the required theory, as well as the experimental setup are presented.

The integration of this optical mechanics experiment adds a new dimension to a mechanical engineering laboratory. The applied stress results obtained using caustics agree with the actual one. One of the secondary objectives of the proposed experiments is help the students to recognise the need for life-long learning. The proposed optical mechanics experiments can be extended to real-life engineering situations where the actual load/stress in polymeric member is unknown.

## NOTATIONS

a	hole radius
A	stress-optic coefficient
C	optical constant
d	thickness of specimen
$D_{max}$	caustic's maximum diameter
E	modulus of elasticity
$L_i$	initial light path
$L_f$	final light path
M	magnification factor
n	refractive index
$R_i$	radius of initial curve
$\nu$	Poisson's ratio
P	applied load
$Z_i$	distance between the divergent light source and the model
$Z_o$	distance between model and screen
W	Plate width
$\sigma$	applied stress

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